DUSO Mathematics League 2014 - 2015

Contest #2. **Calculators are not permitted on this contest.**

Part I. **ALGEBRA I** *Time Limit: 10 minutes*

The word "compute" calls for an exact answer in simplest form.

2 - 1. Compute the ordered pair of numbers (c, d) such that $4c + 19 = -d$ and $-3c + 2d = 28$.

2 - 2. Compute all values of x for which $\dfrac{x^3 + x^2 - 4x - 4}{x^3 - x^2 - 4x + 4} = 0$.

DUSO Mathematics League 2014 - 2015

Contest #2. **Calculators are not permitted on this contest.**

Part II. **GEOMETRY** *Time Limit: 10 minutes*

The word "compute" calls for an exact answer in simplest form.

2 - 3. Suppose that $\overline{PQ}$ has endpoints $P(5, -2)$ and $Q(-3, 10)$. The point R is on $\overline{PQ}$ and divides it such that $PR : RQ = 3 : 1$. Compute the coordinates of R.

2 - 4. The sides of a triangle are in the ratio $8 : 15 : 17$. The area of the triangle (in square inches) is numerically equal to its perimeter (in inches). Compute the length of the longest side (in inches).

DUSO Mathematics League 2014 - 2015

Contest #2. **Calculators are not permitted on this contest.**

Part III. **ALGEBRA II / ADVANCED TOPICS** *Time Limit: 10 minutes*

The word "compute" calls for an exact answer in simplest form.

2 - 5. Compute all solutions to $9^{x^2+8x} = 3^{-24}$.

2 - 6. In $\triangle ABC$, $c = 6$, $b = 8$, $m\angle B = 34.7°$, and $m\angle C = 25.3°$. Compute $b \cos C + c \cos B$.

Author: George Reuter - coachreu@gmail.com – Reviewer: Michael Curry - currymath@gmail.com

DUSO Mathematics League 2014 - 2015

Contest #2. **TEAM ROUND** **Calculators are not permitted on this contest.**

T-1. Suppose that $\sqrt{48 - 2\sqrt{407}}$ can be expressed as $\sqrt{A} - \sqrt{B}$ for counting numbers A and B where $A > B$. Compute $A - B$.

T-2. The three-digit number $N = \underline{ABC}$ is 7 times the two-digit number $\underline{AC}$. Compute the greatest possible value of N. *Note: A, B, and C are digits, so that A is the hundreds digit of N and the tens digit of AC.*

T-3. In $\triangle TRI$, $TR = 4$, $TI = 8$, and $RI = 4\sqrt{3}$. A is on $\overline{TI}$ and N is on $\overline{RI}$ such that $\triangle RAN$ is equilateral. Compute the area of $\triangle RAN$.

Author: George Reuter - coachreu@gmail.com – Reviewer: Michael Curry - currymath@gmail.com

CONTEST #2.

SOLUTIONS

2 - 1. $\boxed{(\mathbf{-6,5})}$ Substitute $d = -4c - 19$ (from the first equation) into the second equation to obtain $-3c + 2(-4c - 19) = 28$. Solving this yields $-11c - 38 = 28 \rightarrow c = -6$. Back-substituting yields $d = 5$. The desired ordered pair is $(\mathbf{-6, 5})$.

2 - 2. $\boxed{\mathbf{-1}}$ Factoring (by grouping?) the numerator and the denominator helps to rewrite the left side of the equation as $\dfrac{(x^2-4)(x+1)}{(x^2-4)(x-1)}$. Recognize that $x = 2$ and $x = -2$ would result in $\frac{0}{0}$, which is indeterminate. Therefore, the only value of x that makes the numerator zero and the denominator non-zero is $x = -1$.

---

2 - 3. $\boxed{(\mathbf{-1,7})}$ The point R is 3/4 of the way from P to Q. Therefore, the x-coordinate is $5 + \frac{3}{4} \cdot -8 = -1$ and the y-coordinate is $-2 + \frac{3}{4} \cdot 12 = 7$. The desired coordinates are $(\mathbf{-1, 7})$.

2 - 4. $\boxed{\mathbf{\frac{34}{3}}}$ The perimeter of the triangle is $40x$ for some x. The triangle is a right triangle (Pythagoras told me so), so its area is $0.5(8x)(15x) = 60x^2$. We equate the two expressions to obtain $60x^2 = 40x$, which solves to give $x = \frac{2}{3}$. Then, we have the largest side as $17 \cdot \frac{2}{3} = \mathbf{\frac{34}{3}}$.

---

2 - 5. $\boxed{\{\mathbf{-2, -6}\} \textit{ must have both}}$ Recognizing that $9 = 3^2$, the given equation may be expressed as $9^{x^2+8x} = 9^{-12}$. Equating exponents, solve
$x^2 + 8x = -12 \rightarrow x^2 + 8x + 12 = 0 \rightarrow (x + 2)(x + 6) = 0$. The solutions are $\{\mathbf{-2, -6}\}$.

2 - 6. $\boxed{\mathbf{2\sqrt{37}}}$ The reader should be convinced (after perhaps drawing in an altitude to $\overline{BC}$) that $b \cos C + c \cos B$ is really just the length a. Note also that $m\angle A = 120°$. Use the Law of Cosines to obtain $a^2 = 8^2 + 6^2 - 2 \cdot 8 \cdot 6 \cdot (-1/2)$, so $a = \sqrt{148} = \mathbf{2\sqrt{37}}$.

Author: George Reuter - coachreu@gmail.com – Reviewer: Michael Curry - currymath@gmail.com

T-1. Suppose that $\sqrt{48 - 2\sqrt{407}}$ can be expressed as $\sqrt{A} - \sqrt{B}$ for counting numbers A and B where $A > B$. Compute $A - B$.

T-1Sol. $\boxed{26}$ Equating the two given expressions and squaring both sides yields $48 - 2\sqrt{407} = A + B - 2\sqrt{AB}$. This gives two equations: $A + B = 48$ and $AB = 407 = 37 \cdot 11$. Therefore, $A = 37$ and $B = 11$. The difference $A - B = \mathbf{26}$.

T-2. The three-digit number $N = \underline{ABC}$ is 7 times the two-digit number $\underline{AC}$. Compute the greatest possible value of N. *Note: A, B, and C are digits, so that A is the hundreds digit of N and the tens digit of AC.*

T-2Sol. $\boxed{\mathbf{105}}$ We have that $100A + 10B + C = 7(10A + C)$, which implies $30A + 10B - 6C = 0 \to 3C = 5(3A + B)$. Since the right side of this last equation is divisible by 5, so is the left, and so $C = 5$. That makes $3A + B = 3$, and thus $A = 1$ and $B = 0$. The only solution has $\mathbf{N} = \mathbf{105}$.

T-3. In $\triangle TRI$, $TR = 4$, $TI = 8$, and $RI = 4\sqrt{3}$. A is on $\overline{TI}$ and N is on $\overline{RI}$ such that $\triangle RAN$ is equilateral. Compute the area of $\triangle RAN$.

T-3Sol. $\boxed{\mathbf{3\sqrt{3}}}$ Note that $\triangle RAN$ is equilateral, so its area is given by $\dfrac{s^2\sqrt{3}}{4}$, where $s = RA$. Note also that $m\angle ARN = 60^\circ$, so $m\angle TRA = 30^\circ$, and thus $m\angle TAR = 90^\circ$. Therefore, $RA = 4\sin 60^\circ = 2\sqrt{3}$. Now, the area of $\triangle RAN = (2\sqrt{3})^2\dfrac{\sqrt{3}}{4} = \mathbf{3\sqrt{3}}$.

Author: George Reuter - coachreu@gmail.com – Reviewer: Michael Curry - currymath@gmail.com